Mathematics Strategy

Procedural Fluency Developed from Conceptual Understanding

What is conceptual understanding and what is procedural fluency?

Conceptual Understanding refers to the ability to explain how mathematical operations or procedures relate to a physical context or process, how mathematical rules or procedures are derived, and how operations and procedures relate to each other. Conceptual understanding allows students to use procedures in a flexible way and to adapt their use to novel situations.

Procedural fluency refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

How do teachers develop procedural fluency from conceptual understanding?

"Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skilful in using procedures flexibly as they solve contextual and mathematical problems." (*NCTM*, 2014).Teaching through problem solving is a great way to get students making sense of concepts. Posing problems at the beginning of a topic and encouraging students to find solutions can lead to mathematical generalizations and procedures that are grounded in student reasoning.

Building procedural fluency from conceptual understanding requires a thorough understanding of relevant conceptual models and structures. For example, thinking about multiplication simply as repeated addition does not help when students encounter questions such as $\frac{1}{2}\times\frac{-3}{4}$. Understanding multiplication as repeated addition, arrays or grids , measurement (the area of a rectangle with dimensions 3 cm and 5 cm), ratios or rates (e.g. 5 kg at $3/kg), and other models, supports students in having conceptual understanding to support when and why a multiplication procedure works. Procedural fluency that has a strong foundation in conceptual understanding allows students to transfer procedures to various context accurately.

Teachers and students have a role in developing procedural fluency from conceptual understanding. Table 1 outlines teacher actions and student actions that support this learning.

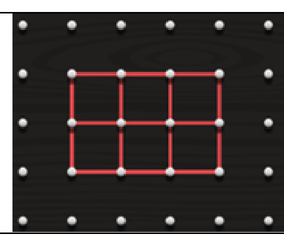
Table 1

Procedural Fluency Developed from Conceptual Understanding Teacher and Student Actions	
What are teachers doing?	**What are students doing?**
Providing students with opportunities to use their own reasoning strategies and methods for solving problems.	Making sure that they understand and can explain the mathematical basis for the procedures that they are using.
Asking students to discuss and explain why the procedures that they are using work to solve particular problems.	Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.
Connecting student-generated strategies and methods to more efficient procedures as appropriate.	Determining whether specifies approaches generalize to a broad class of problems.

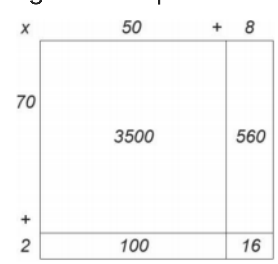
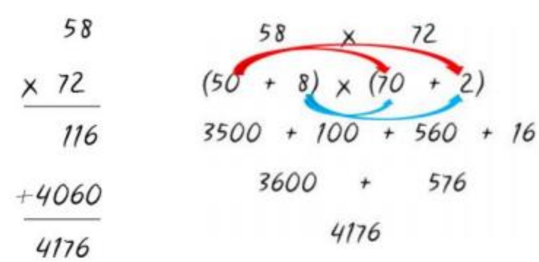
(from: NCTM Principles to Actions: Ensuring Mathematical Success for All, 2014)

Examples

Example 1	**What is 25 % of 84**
Conceptual Understanding: *comprehension of mathematical concepts, operations, and relations*	A student with strong conceptual understanding might recall that 25% is the same as ¼ , and be able to use mental math to find the result, understanding that “25% of”, “one-fourth of”, multiplying by 0.25, and dividing by 4, are all different ways to understand this one problem
Procedural Fluency: *the ability to apply procedures accurately, efficiently, and flexibly*	A student with procedural knowledge and fluency can generalize their conceptual knowledge to the procedure of finding the product of 0.25 x 84 = 21 to solve the problem. This procedure could be practiced to improve the efficiency of solving this common type of problem.

Example 2	

Conceptual Understanding: *Comprehension of mathematical concepts, operations, and relations*	Students understand conceptually that the two by three rectangle above has an area of 6 square units, without the use of an area formula or algorithm. The idea or *concept* that area is measured in square units is supported by the visual.
Procedural Fluency: *the ability to apply procedures accurately, efficiently, and flexibly*	Students recognize the multiplicative relationship between the side lengths and the area and use this observation to identify a procedure for efficiently computing the area of any rectangle.

Example 3	**What is 58 × 72?**
Conceptual Understanding: *Comprehension of mathematical concepts, operations, and relations*	Students can understand that conceptually multiplication expressions are geometric and can be represented using a conceptual model such as an open array. 
Procedural Fluency: *the ability to apply procedures accurately, efficiently, and flexibly*	Students recognize the multiplicative relationship between the side lengths and the area and use this observation to identify a procedure for efficiently computing the area of any rectangle which is the product of the multiplication expression. 

Additional Student Computation Work Samples that outline mental math strategies, models to support conceptual understanding and procedures can be found on the math strategy insite page.

Resources:

The following sources provide detailed examples of *building procedural fluency from conceptual understanding*:

References:

National Council of Teachers of Mathematics (2014)., *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA:NCTM

Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education, National Research Council (2001). *Adding It Up: helping children learn mathematics.* Washington, DC: National Academy Press

John Hattie., Douglas Fisher., Dr. Nancy., Linda M Gojak.,Sara Delano Moore., William L Mellman. (2016). Visible *Learning for Mathematics, Grades K-12: What Works Best to Optimize Student Learning.* Corwin. A SAGE company.

Van de Walle, J., Karp, S., Bay-Williams, J., & McGarvey, L. (2017). *Elementary and middle school mathematics teaching developmentally*. Don Mills, ON: Pearson Canada Inc.